UNIT IV

DIGITAL MODULATION SCHEME

Geometric Representation of Signals

- Objective: To represent any set of ***M*** *energy signals* $\{s_i(t)\}$ as linear combinations of ***N*** *orthogonal basis functions*, where N ≤ M
- Real value energy signals $s_1(t), s_2(t),..s_M(t)$, each of duration T

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \quad \begin{Bmatrix} 0 \le t \le T \\ i==1,2,\ldots,M \end{Bmatrix} \qquad 4.1$$

Orthogonal basis function

coefficient

Energy signal

- Coefficients:

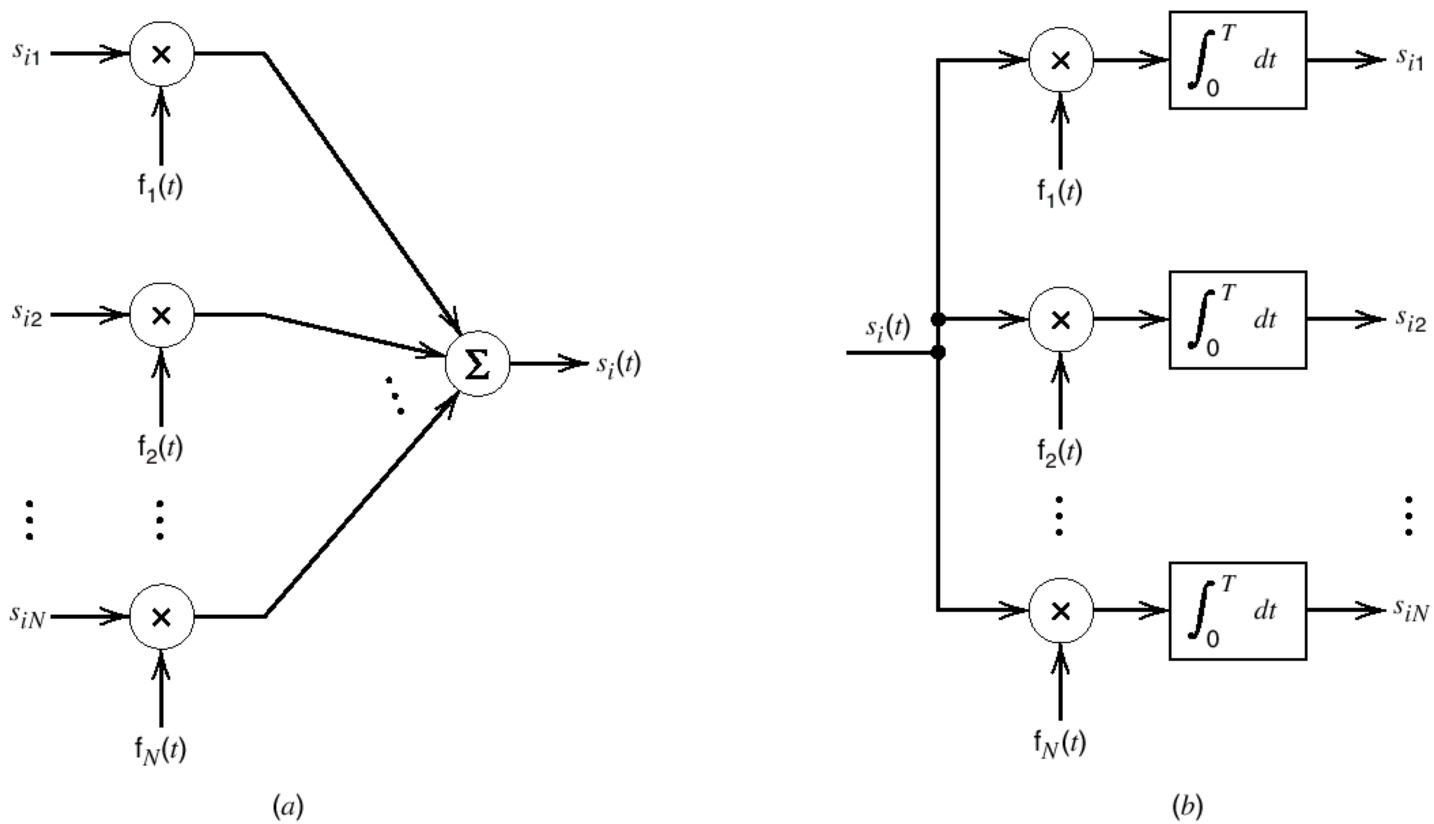
$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt, \qquad \begin{Bmatrix} \text{i=1,2,....,M} \\ \text{j=1,2,....,M} \end{Bmatrix}$$

- Real-valued basis functions:

$$\int_0^T \phi_i(t)\phi_j(t)dt = \delta_{ij} = \begin{Bmatrix} 1 \ \text{ if } i = j \\ 0 \ \text{ if } i \neq j \end{Bmatrix}$$

- The set of coefficients can be viewed as a N-dimensional vector, denoted by s_i
- Bears a one-to-one relationship with the transmitted signal $s_i(t)$

(a) Synthesizer for generating the signal $s_i(t)$. (b) Analyzer for generating the set of signal vectors $\{s_i\}$.



So,

- Each signal in the set $s_i(t)$ is completely determined by the vector of its coefficients

$$\mathrm{s_i} = \begin{bmatrix} s_{i1} \\ s_{i2} \\ . \\ . \\ . \\ s_{iN} \end{bmatrix}, \qquad i = 1,2,\ldots.,\mathrm{M}$$

Finally,

- The signal vector $\boldsymbol{s_i}$ concept can be extended to 2D, 3D etc. N-dimensional Euclidian space
- Provides mathematical basis for the geometric representation of energy signals that is used in noise analysis
- Allows definition of
  - Length of vectors (absolute value)
  - Angles between vectors
  - Squared value (inner product of s_i with itself)

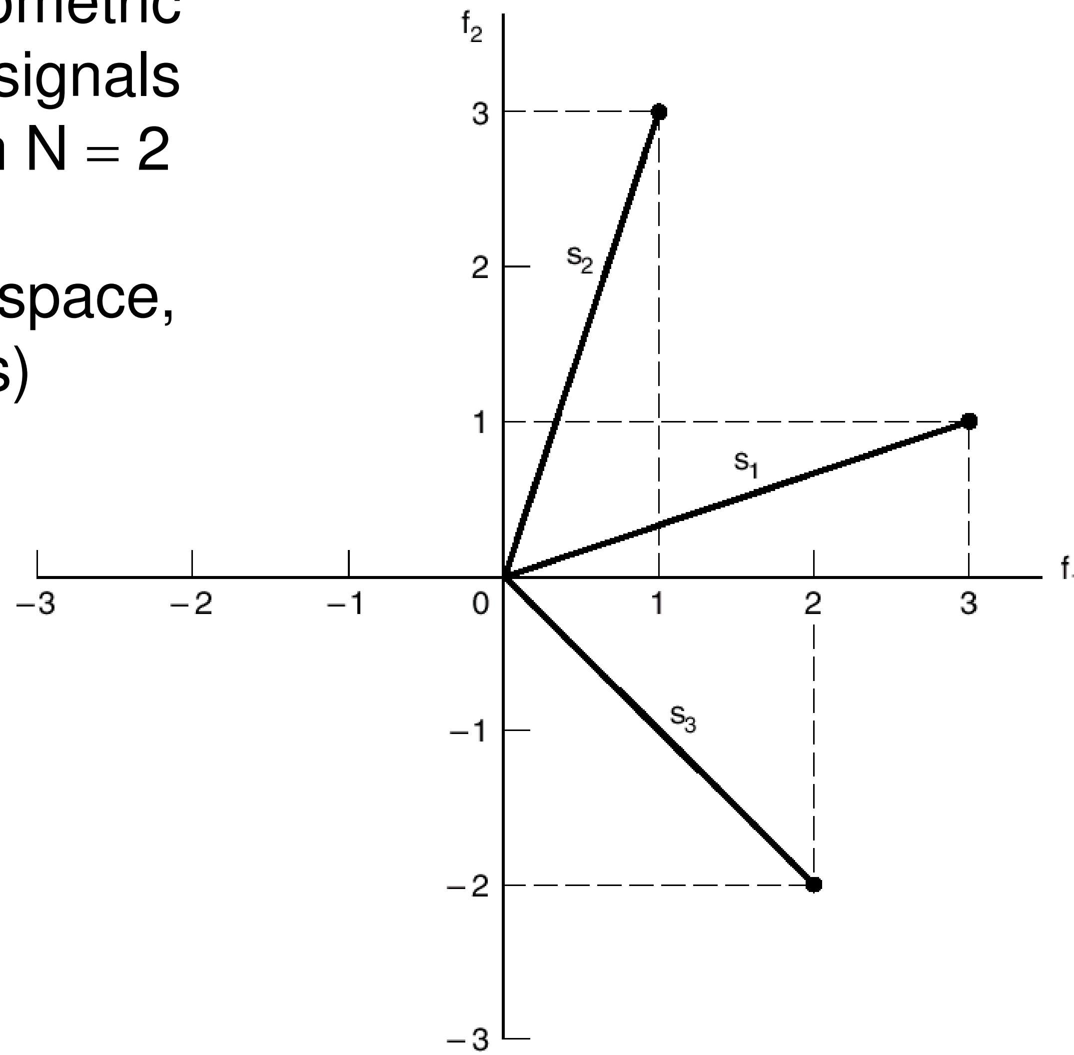
$$\|\mathrm{s_i}\|^2 = s_i^T \mathrm{s_i}$$

Matrix Transposition

$$= \sum_{j=1}^{N} s_{ij}^2, \qquad i = 1,2,\ldots,\mathrm{M}$$

Figure 5.4

Illustrating the geometric representation of signals for the case when N = 2 and M = 3.
(two dimensional space, three signals)



Also,

What is the relation between the *vector representation* of a signal and its *energy value?*

- …start with the definition of average energy in a signal…

$$E_i = \int_0^T s_i^2(t)\,dt$$

- Where $s_i(t)$ is

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t),$$

- After substitution: $$E_i = \int_0^T \left[\sum_{j=1}^{N} s_{ij} \phi_j(t) \right] \left[\sum_{k=1}^{N} s_{ik} \phi_k(t) \right] dt$$

- After regrouping: $$E_i = \sum_{j=1}^{N} \sum_{k=1}^{N} s_{ij} s_{ik} \int_0^T \phi_j(t) \phi_k(t) dt$$

- $\Phi_j(t)$ is orthogonal, so finally we have: $$E_i = \sum_{j=1}^{N} s_{ij}^2 = \| s_i \|^2$$

The *energy of a signal* is equal to the *squared length of its vector*

Formulas for two signals

- Assume we have a pair of signals: $s_i(t)$ and $s_j(t)$, each represented by its vector,
- Then:

$$s_{ij} = \int_0^T s_i(t) s_k(t) dt = s_i^T \mathrm{s}_\mathrm{k}$$

Inner product of the signals is equal to the inner product of their vector representations [0,T]

Inner product is invariant to the selection of basis functions

Euclidian Distance

- The Euclidean distance between two points represented by vectors (signal vectors) is equal to

  $||s_i - s_k||$ and the squared value is given by:

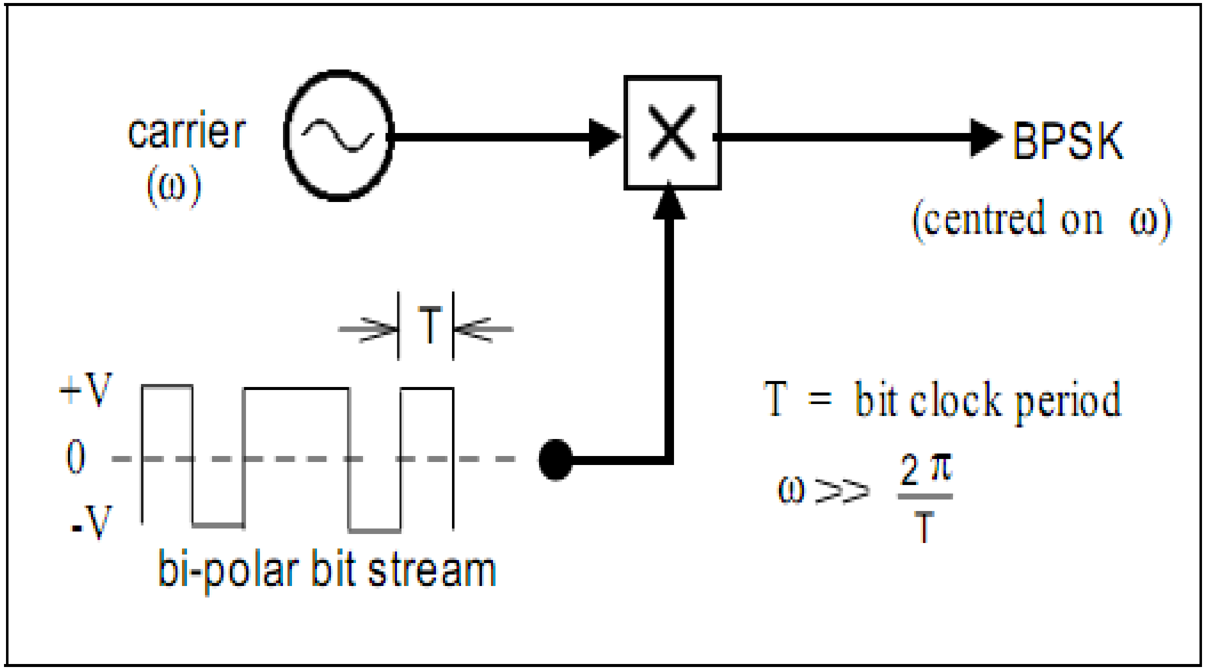
$$\|\mathrm{s}_{\mathrm{i}} - \mathrm{s}_k\|^2 = \sum_{j=1}^{N} (s_{ij} - s_{kj})^2$$

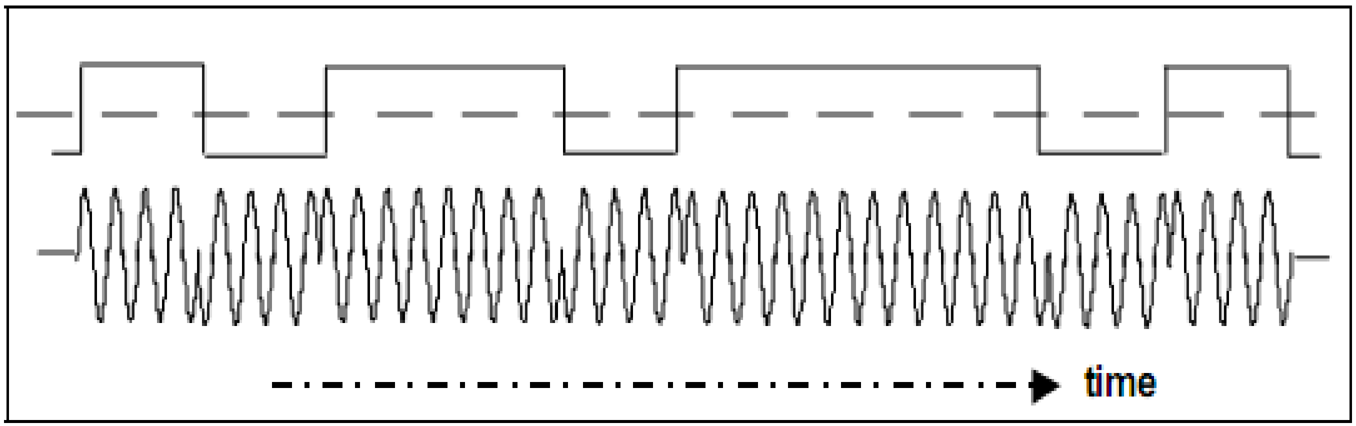
$$= \int_0^T (s_i(t) - s_k(t))^2 \, dt$$

BPSK - BINARY PHASE SHIFT KEYING

Generation of BPSK:

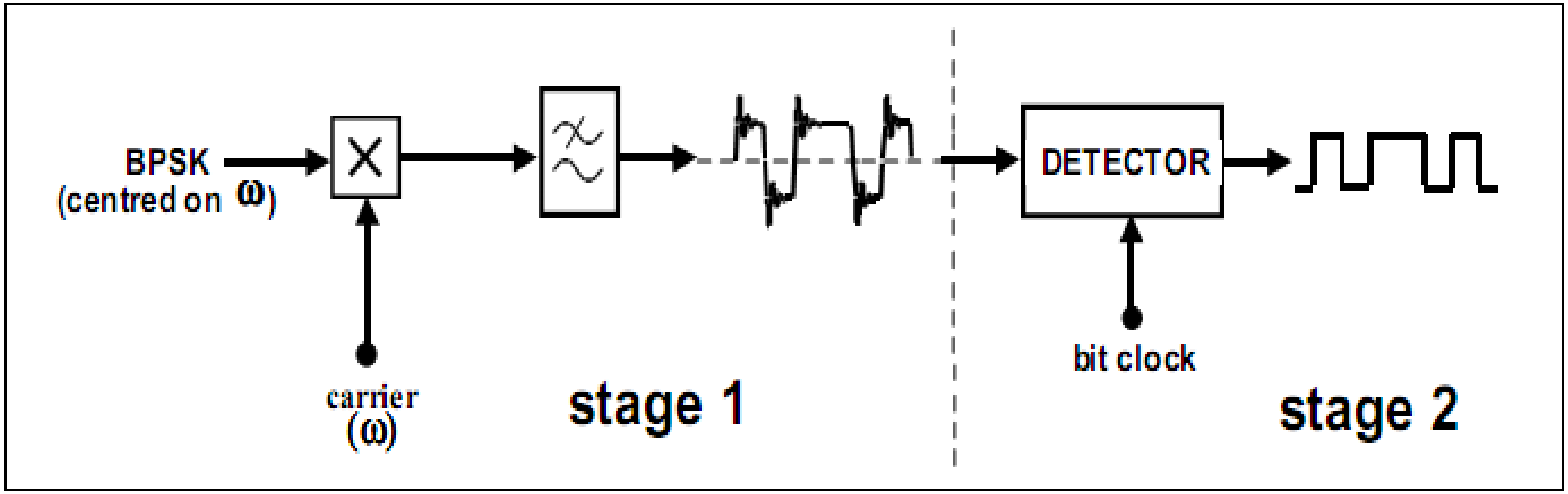
- Consider a sinusoidal carrier. If it is modulated by a bi-polar bit stream, its polarity will be reversed every time the bit stream changes polarity.
- This, for a sinewave, is equivalent to a phase reversal (shift). The multiplier output is a BPSK 1 signal.


carrier
(ω)
BPSK
(centred on ω)
T
+V
0
-V
bi-polar bit stream
T = bit clock period
ω >> 2π/T



BPSK signal in time domain

sychronous demodulation of BPSK



Frequency Shift Keying (FSK)

- Binary FSK

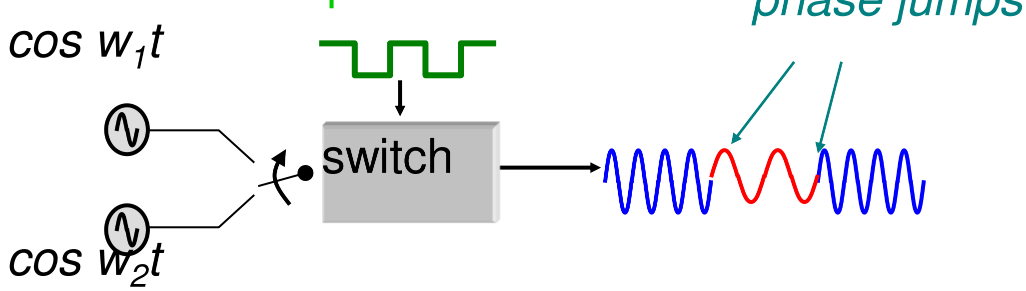
- Frequency of the constant amplitude carrier is changed according to the message state → high (1) or low (0)

$$s_1(t) = A\cos(2\pi f_c + 2\pi\Delta f)t \quad 0 \le \mathrm{t} \le \mathrm{T_b} \ (\mathrm{bit} = 1)$$
$$s_2(t) = A\cos(2\pi f_c - 2\pi\Delta f)t \quad 0 \le \mathrm{t} \le \mathrm{T_b} \ (\mathrm{bit} = 0)$$

- Discontinuous / Continuous Phase

Discontinuous Phase FSK

Switching between 2 independent oscillators for binary *1* & *0*



binary 1→ $s_{BFSK}(t) = v_H(t)$

$$= \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_H t + \theta_1) \qquad 0 \le t \le T_b$$

binary 0→ $s_{BFSK}(t) = v_L(t)$

$$= \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_L t + \theta_2) \qquad 0 \le t \le T_b$$

- **results in phase discontinuities**
- **discontinuities causes spectral spreading & spurious transmission**
- **not suited for tightly designed systems**

Continuous Phase FSK

single carrier that is frequency modulated using *m(t)*

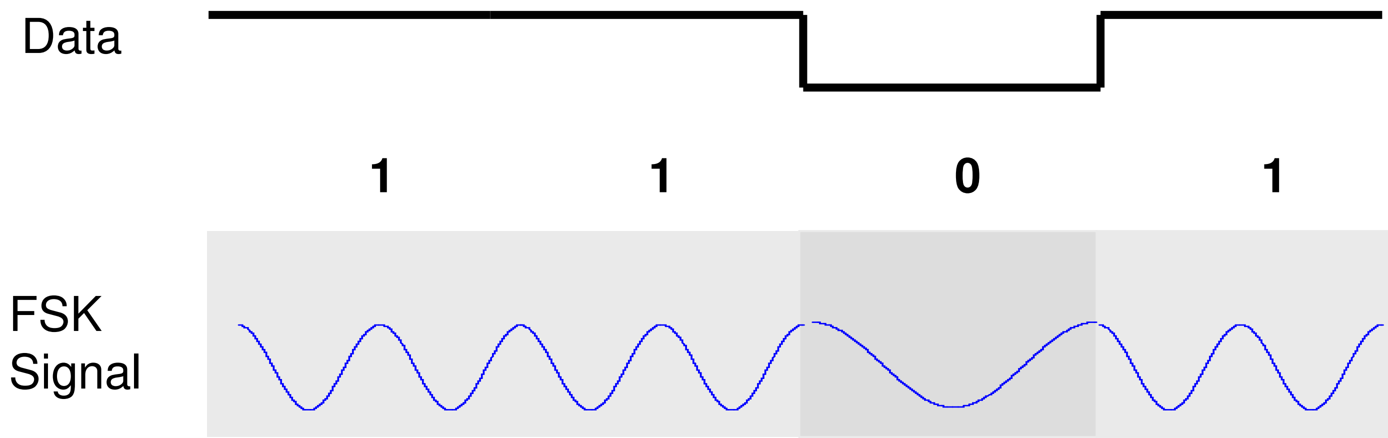
$$s_{BFSK}(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta(t))$$

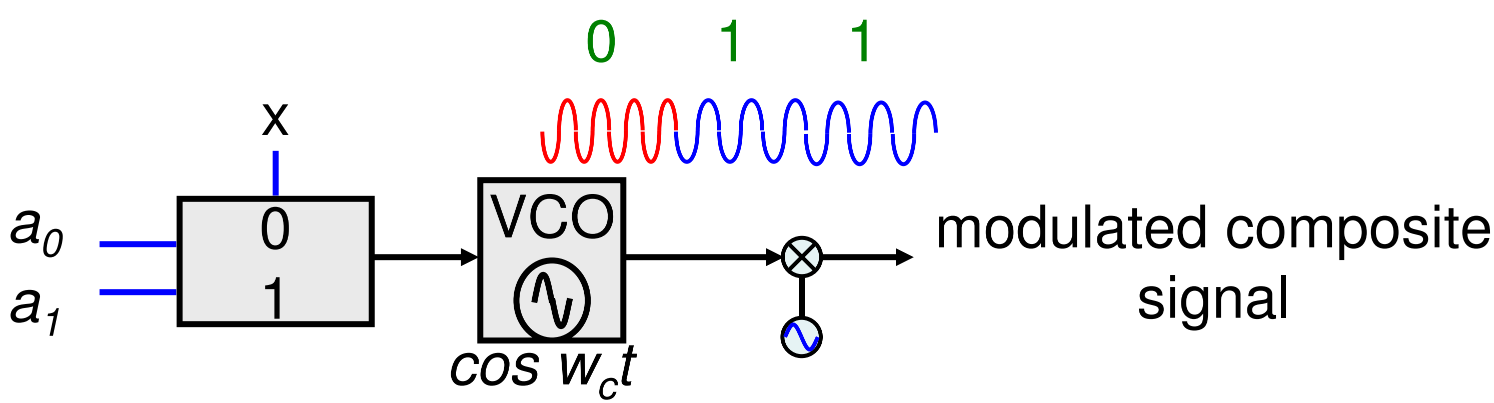
$$= \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t + 2\pi k_{FSK} \int_{-\infty}^{t} m(\eta)\, d\eta\right)$$

where $\theta(t) = 2\pi k_{FSK} \int_{-\infty}^{t} m(\eta)\, d\eta$

- *m(t)* = discontinuous bit stream
- *θ(t)* = **continuous phase** function proportional to integral of *m(t)*

FSK Example


Data
1
1
0
1
FSK
Signal


0
1
1
x
a_0
a_1
0
1
VCO
$cos\ w_c t$
modulated composite signal

Spectrum & Bandwidth of BFSK Signals

- complex envelope of BFSK is nonlinear function of *m(t)*
- spectrum evaluation - difficult - performed using actual time averaged measurements

PSD of BFSK consists of discrete frequency components at

- f_c
- $f_c \pm n\Delta f$, *n is an integer*

PSD decay rate (inversely proportional to spectrum)

- PSD decay rate for CP-BFSK → $\frac{1}{\Delta f^4}$
- PSD decay rate for non CP-BFSK → $\frac{1}{\Delta f^2}$

Δf = frequency offset from f_c

Spectrum & Bandwidth of BFSK Signals

Transmission Bandwidth of BFSK Signals (from Carson's Rule)

- B = bandwidth of digital baseband signal
- B_T = transmission bandwidth of BFSK signal

$$B_T = 2\Delta f + 2B$$

- assume 1st null bandwidth used for digital signal, B

  - bandwidth for **rectangular pulses** is given by $B = R_b$
  - bandwidth of BFSK using **rectangular pulse** becomes

$$B_T = 2(\Delta f + R_b)$$

if **RC pulse** shaping used, bandwidth reduced to:

$$B_T = 2\Delta f + (1+\alpha)\, R_b$$

General FSK signal and orthogonality

- Two FSK signals, $V_H(t)$ and $V_L(t)$ are **orthogonal** if

$$\int_0^T V_H(t)V_L(t)dt = 0$$

?

- interference between $V_H(t)$ and $V_L(t)$ will average to 0 during demodulation and integration of received symbol

- received signal will contain $V_H(t)$ and $V_L(t)$

- demodulation of $V_H(t)$ results in $(V_H(t) + V_L(t))\,V_H(t)$

$$\int_0^T V_H(t)V_L(t)dt = 0 \qquad \int_0^T V_H(t)V_H(t)dt \neq 0$$

?

An FSK signal for $0 \le t \le T_b$

$v_H(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi(f_c + \Delta f)t)$ and $v_L(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi(f_c - \Delta f)t)$

then $v_H(t)\, v_L(t) =$

$$\frac{2E_b}{T_b}\cos(2\pi(f_c + \Delta f)t)\cos(2\pi(f_c - \Delta f)t)$$

$$= \frac{E_b}{T_b}\left[\cos(2\pi(2f_c)t) + \cos(2\pi(2\Delta f)t)\right]$$

and

$$\int_0^T V_H(t)V_L(t)dt = \int_0^{T_b}\frac{E_b}{T_b}\left[\cos(4\pi f_c t) + \cos(4\pi\Delta f t)\right]dt$$

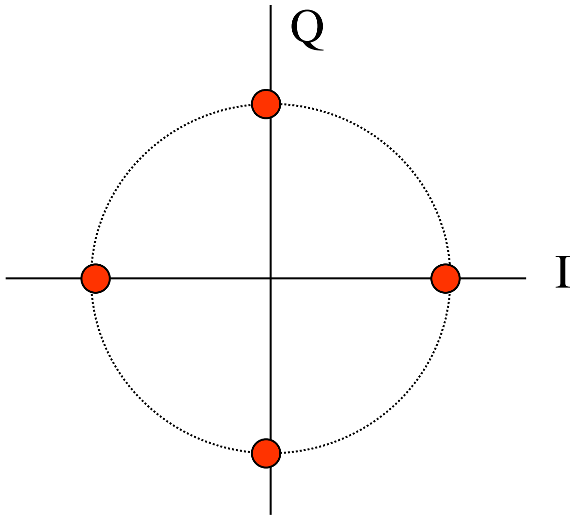
$$= \frac{E_b}{T_b}\left[\frac{\sin(4\pi f_c t)}{4\pi f_c} + \frac{\sin(4\pi\Delta f t)}{4\pi\Delta f}\right]_0^{T_b} = \frac{E_b}{T_b}\left[\frac{\sin(4\pi f_c T_b)}{4\pi f_c} + \frac{\sin(4\pi\Delta f T_b)}{4\pi\Delta f}\right]$$

$v_H(t)\, v_L(t)$ are orthogonal if $\Delta f \sin(4\pi f_c T_b) = -f_c(\sin(4\pi\Delta f\, T_b)$

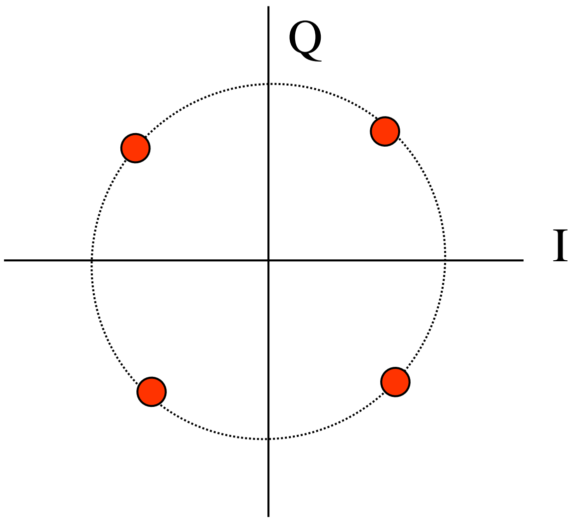
QPSK

- Quadrature Phase Shift Keying (QPSK) can be interpreted as two independent BPSK systems (one on the I-channel and one on Q-channel), and thus the same performance but twice the bandwidth (spectrum) efficiency.

QPSK Constellation Diagram



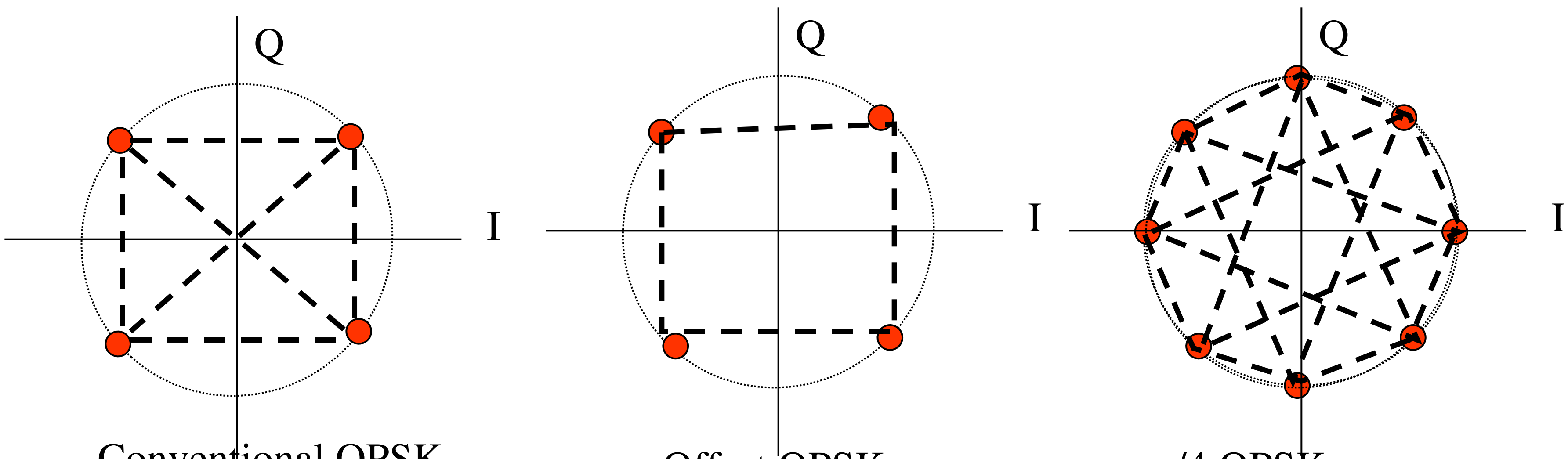
Carrier phases
$\{0, \pi/2, \pi, 3\pi/2\}$



Carrier phases
$\{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$

- Quadrature Phase Shift Keying has twice the bandwidth efficiency of BPSK since 2 bits are transmitted in a single modulation symbol

Types of QPSK



Conventional QPSK   Offset QPSK $\pi/4$ QPSK

- Conventional QPSK has transitions through zero (i.e. 180^0 phase transition). Highly linear amplifiers required.
- In Offset QPSK, the phase transitions are limited to 90^0, the transitions on the I and Q channels are staggered.
- In $\pi/4$ QPSK the set of constellation points are toggled each symbol, so transitions through zero cannot occur. This scheme produces the lowest envelope variations.
- All QPSK schemes require linear power amplifiers

Quadrature Phase Shift Keying (QPSK):

•Also a type of linear modulation scheme

•Quadrature Phase Shift Keying (QPSK) has twice the bandwidth efficiency of BPSK, since 2 bits are transmitted in a single modulation symbol.

• The phase of the carrier takes on 1 of 4 equally spaced values, such as where each value of phase corresponds to a unique pair of message bits.

• The QPSK signal for this set of symbol states may be defined as:

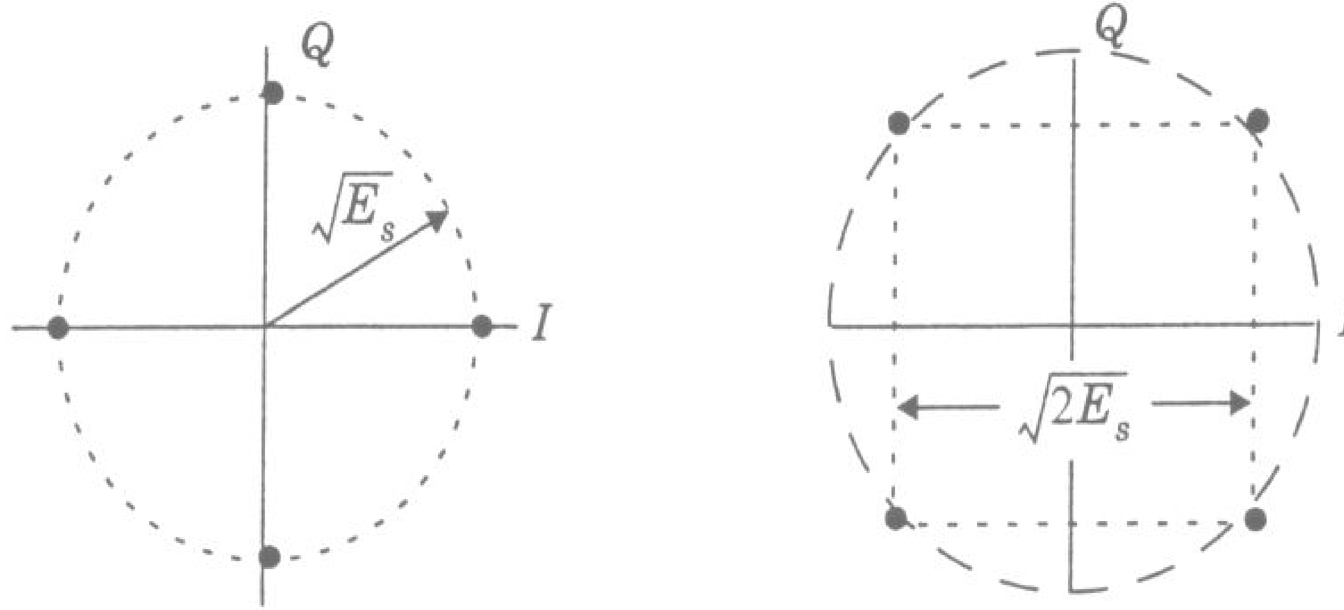
$$0,\ \pi/2,\ \pi,\ \text{and}\ 3\pi/2,$$

$$S_{QPSK}(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left[2\pi f_c t + (i-1)\frac{\pi}{2}\right] \quad 0 \le t \le T_s \quad i = 1, 2, 3, 4.$$

QPSK

$$S_{QPSK} = \left\{ \sqrt{E_s}\cos\left[(i-1)\frac{\pi}{2}\right]\phi_1(t) - \sqrt{E_s}\sin\left[(i-1)\frac{\pi}{2}\right]\phi_2(t) \right\} \quad i = 1, 2, 3, 4$$

- The striking result is that the bit error probability of QPSK is identical to BPSK, but twice as much data can be sent in the same bandwidth. Thus, when compared to BPSK, QPSK provides **twice the spectral efficiency** with exactly the same energy efficiency.
- Similar to BPSK, QPSK can also be differentially encoded to allow non-coherent detection.



(a)

(b)

(a) QPSK constellation where the carrier phases are $0, \pi/2, \pi, 3\pi/2$.
(b) QPSK constellation where the carrier phases are $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$.

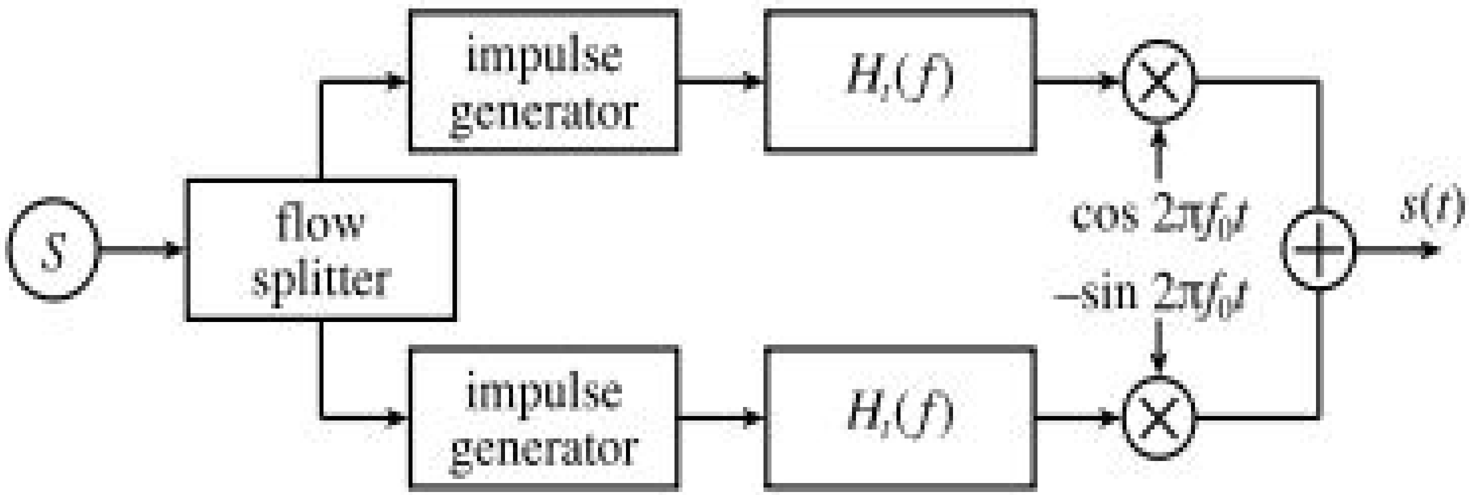
Quadrature amplitude modulation

- **Quadrature amplitude modulation** (**QAM**) is both an analog and a digital modulation scheme.

- It conveys two analog message signals, or two digital bit streams, by changing (*modulating*) the amplitudes of two carrier waves, using the amplitude-shift keying(ASK) digital modulation scheme or amplitude modulation (AM) analog modulation scheme.

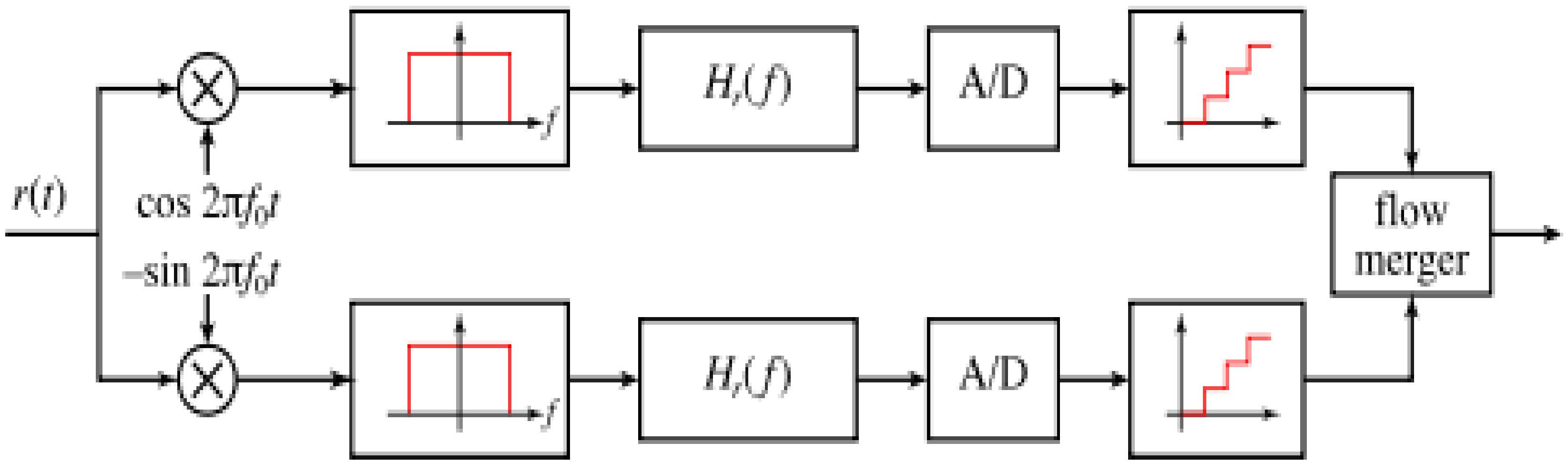
- The two carrier waves, usually sinusoids, are out of phase with each other by 90° and are thus called quadrature carriers or quadrature components — hence the name of the scheme.

QAM Transmitter



- First the flow of bits to be transmitted is split into two equal parts: this process generates two independent signals to be transmitted.
- They are encoded separately just like they were in an amplitude-shift keying (ASK) modulator.
- Then one channel (the one "in phase") is multiplied by a cosine, while the other channel (in "quadrature") is multiplied by a sine.
- This way there is a phase of 90° between them. They are simply added one to the other and sent through the real channel.

QAM Receiver



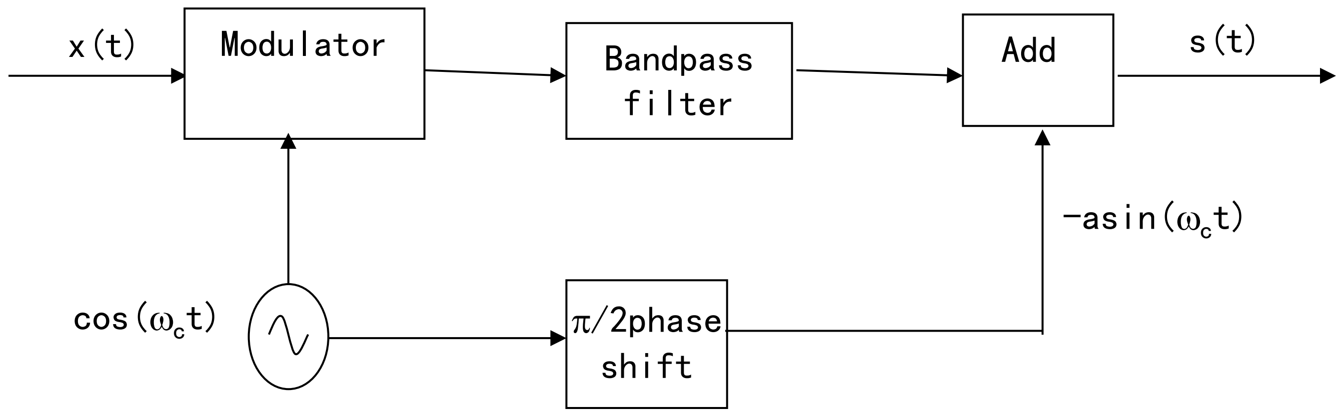
- The receiver simply performs the inverse operation of the transmitter.
- Multiplying by a cosine (or a sine) and by a low-pass filter it is possible to extract the component in phase (or in quadrature).
- Then there is only an ASK demodulator and the two flows of data are merged back.

Carrier Synchronization

- Synchronization is one of the most critical functions of a communication system with coherent receiver. To some extent, it is the basis of a synchronous communication system.
- Carrier synchronization
- Symbol/Bit synchronization
- Frame synchronization

- Receiver needs estimate and compensate for frequency and phase differences between a received signal’s carrier wave and the receiver’s local oscillator for the purpose of coherent demodulation, no matter it is analog or digital communication systems.
- To extract the carrier：
- 1. Pilot-tone insertion method
- Sending a carrier component at specific spectral-line along with the signal component. Since the inserted carrier component has high frequency stability, it is called pilot.
- 2. Direct extraction method
- Directly extract the synchronization information from the received signal component.

1. Pilot-tone insertion method —insert pilot to the modulated signal



- The pilot signal is generated by shift the carrier by 90^0 and decrease by several dB, then add to the modulated signal. Assume the modulated signal has 0 DC component, then the pilot is

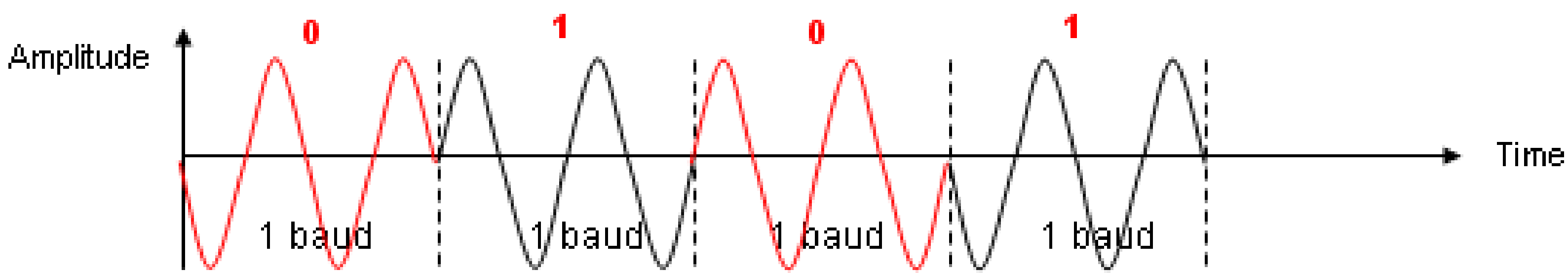
$$s(t) = f(t)\cos\omega_c t - a\sin\omega_c t$$

2. Direct extraction method

- If the spectrum of the received signal already contains carrier component, then the carrier component can be extracted simply by a narrowband filter or a PLL.
- If the modulated signal supresses the carrier component, then the carrier component may be extracted by performing nonlinear transformation or using a PLL with specific design

DPSK

- **DPSK** is a kind of phase shift keying which avoids the need for a coherent reference signal at the receiver.
- Differential BPSK
  - 0 = same phase as last signal element
  - 1 = 180º shift from last signal element



DPSK modulation and demodulation

- 3dB loss

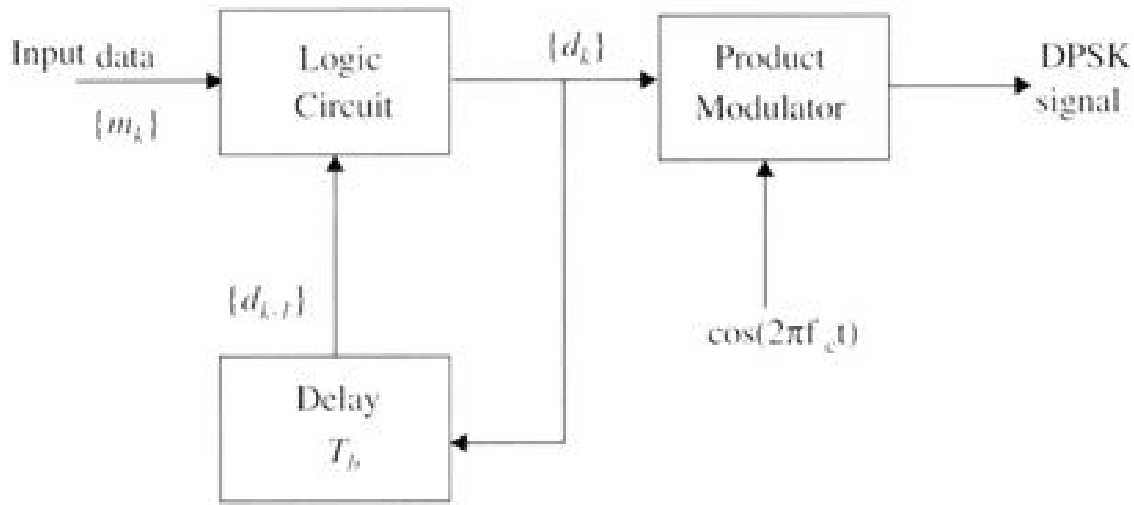
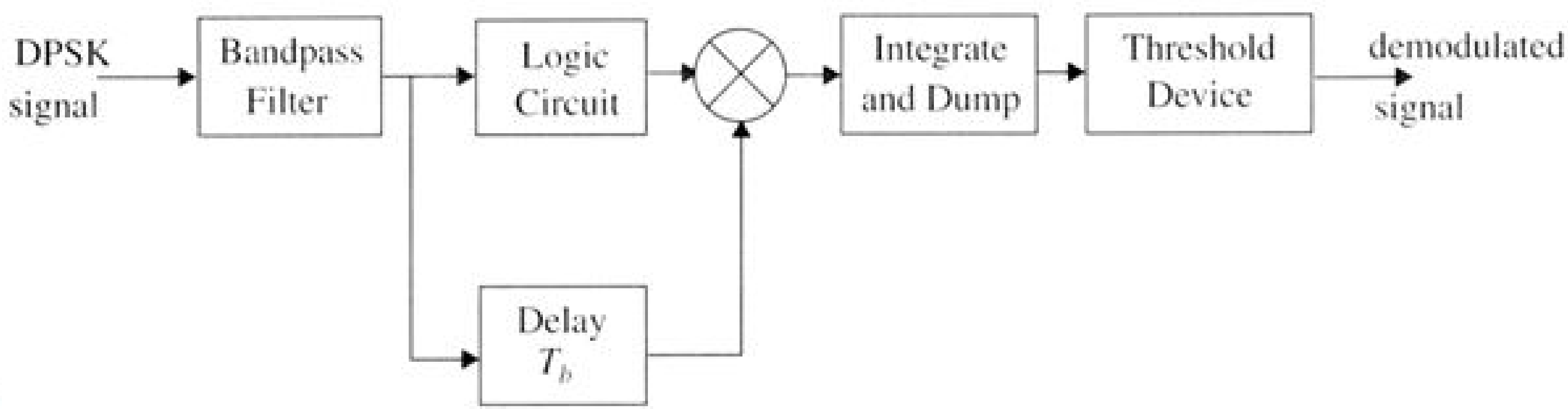


Figure 6.24 Block diagram of a DPSK transmitter.



Thank you